Reduced order calculation method of subsynchronous oscillation mode of PMSG based wind farm connected with power grid based on Arnoldi method

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Abstract. Large scale wind turbine connected to the power grid is easy to induce subsynchronous oscillation. “Curse of dimensionality” will be encountered when the subsynchronous oscillation mode is analysed and evaluated based on the full-order model of the wind farm. It is urgent to research the fast and effective analysis method of subsynchronous oscillation mode when the wind farm is connected to the power grid. Therefore, a reduced order calculation method of the subsynchronous oscillation mode of PMSG based wind farm connected with a power grid based on the Arnoldi method is proposed. This method can quickly obtain the eigenvalues corresponding to the subsynchronous oscillation mode through iterative approximation when PMSG-based wind farm connected to the power system. This can be used to analyze the subsynchronous oscillation of large-scale wind farms connected to the power grid and provide an effective means for the evaluation of it. By changing system stability, collector network structure, and wind farm interconnection scale, the accuracy of the eigenvalues distribution between the full-order system and the reduced order system are compared. Moreover, the reduced order and computation time are considered. The results present that the proposed method can quickly and accurately obtain the subsynchronous oscillation mode in the system.

Keywords: PMSG, Subsynchronous oscillation, Eigenvalue analysis, Model order reduction, Arnoldi method.

1 Introduction

Developing and utilizing wind power is an important step to help achieve decarbonization goals in the energy sector. The cumulative installed capacity of wind power in China has reached 330 million kilowatts by the end of 2021, meanwhile, it is expected to reach 2.4 billion kilowatts by the end of 2050, indicating a rapid development trend in wind power generation [1–3]. However, as the scale of wind power grid connections increases, sub synchronous oscillation accidents in wind power grid connections occur frequently [4–6]. In 2015, an SSO accident occurred at a direct drive wind farm in Hami, Xinjiang, when it was connected to the Xinjiang power grid via a long-distance nonseries compensation line, posing a serious threat to the safe operation of the power system [7–9].

The eigenvalue method is an effective method for analyzing the SSO problem of wind farm interconnection. However, due to the characteristics of wind farms such as a large number of units, diverse operating conditions, and high model orders, conducting eigenvalue analysis on the full-order model of wind farms often encounters the problem of “curse of dimensionality”. Therefore, it is an inevitable trend to reduce the full-order model reasonably. Common simplified methods include aggregation modeling and model reduction techniques.

Aggregation modeling aims to reduce the order of wind farm models by establishing single or multi-machine equivalent models. The single machine equivalent model refers to equating the entire wind farm into one unit, with the research goal of obtaining equivalent unit parameters that can improve model accuracy. Common methods include the capacity weighting method [10–12] and the parameter identification method [13, 14]; however, when there are significant differences in the operating conditions of the fans on-site, the accuracy of the model is difficult to ensure. Multi-machine equivalence refers to dividing the wind farm units into several clusters based on certain indicators and equating them into one unit [15–17], and characterizing the entire wind farm into the form of parallel connection of each equivalent unit; the focus of the study is on the determination of clustering indicators [18, 19]. Multi-machine equivalence has good applicability for scenarios with complex working conditions, but as for large wind
farms, obtaining clustering indicators for each unit and forming an aggregation model after changing the operating point will consume a lot of additional computing resources, even exceeding the feature value calculation of the detailed model.

Model reduction technique means using rigorously proven mathematical methods to simplify the state space matrix of the target system and obtain a reduced order model that meets the requirements of the problem [20]. Common reduction methods include but are not limited to Krylov subspace method, balanced truncation method, and orthogonal decomposition method. Compared to aggregation modeling, this type of method takes the state space equation of the full order system as the object, and the required indicators can be obtained from the state space matrix itself, eliminating the need for additional operations on wind turbines from the physical level. The balanced truncation method is used to obtain a reduced order wind farm model in [21, 22], which can maintain a low-frequency response, for the subsequent design and tuning of relevant controllers. A simulation of doubly fed wind farms connected to DC transmission systems is constructed in [23]. The singular value decomposition method, balanced truncation method, Krylov dynamic subspace method, and related improved algorithms are used to reduce the order of wind farms of different scales. The effectiveness of the reduced order model is verified by comparing the retention of key characteristic values, frequency response, and time-domain simulation before and after the reduction. The study in [24] proposes a singular perturbation reduction method for direct-drive wind farms based on dominance analysis, with the ultimate goal of reproducing the dynamic characteristics and sub-synchronous oscillation characteristics of the full-order system, to achieve the reduction of the direct-drive wind farm system. The complete model of an inverter-type distributed power source is divided into multiple time scale features and ignores fast dynamic order reduction [25]. In the field of wind farm-connected SSO, research on the reduction of state space equations is mostly based on the principle of state matrix similarity transformation, and there is little attention paid to the model reduction techniques [26–28]. Although the principle of similarity transformation solves the “curse of dimensionality” problem in eigenvalue analysis, it often assumes that the operating conditions of the units are similar. In addition, the units are all parallel to the outlet busbar, which makes it difficult to fully consider the differences in operating conditions of the units in the field. On the other hand, the differences in the collector network have significant limitations. The model reduction technique is a universal method for linear systems, which has no physical structural requirements for wind farms and is theoretically applicable to various wind farms with collector networks.

In summary, this article proposes a reduced order calculation method for PMSG wind farm connected SSO mode based on the Arnoldi method (subordinate to the Krylov subspace method). The Arnoldi method, as a partial eigenvalue calculation technique, has been used in traditional power systems to reduce the order and solve the key eigenvalues of low-frequency oscillations and has successfully solved the problem of “curse of dimensionality” in low-frequency oscillations [29–31]. This article modifies it to quickly calculate the current SSO mode in the scenario of a large PMSG wind farm connected to a weak grid. Firstly, this article establishes a full-order state space model for wind farms. Secondly, the general process of model reduction technology and the basic principle of the Arnoldi method were elucidated, and their modification was made to address the SSO problem of wind farm networking. Finally, in the Matlab environment, a wind farm interconnection calculation system was constructed. By changing system stability, collector network structure, and wind farm interconnection scale, the accuracy of the eigenvalue distribution between the full order system and the reduced order system are compared. Moreover, the reduced order and computation time are considered. The results verified the effectiveness, applicability, and speed of the method.

2 Linear modeling of PMSG networked system

2.1 PMSG wind farm networking structure

The PMSG wind farm networking system is shown in Figure 1, which includes M parallel branches and a total of N PMSGs; each branch is composed of several units with the same parameters and control strategies, which are connected to the collector network through step-up transformers. Each branch is connected in parallel to the 35 kV busbar at the wind farm outlet and is boosted to 220 kV through a transformer at the wind farm outlet before being connected to the AC power grid through the line.

In Figure 1, $L_{ck}$ ($k = 1, 2, ..., N$) is the equivalent inductance of the collector line connected to the k-th unit; $L_s$ is the equivalent inductance of the generator transformer, the equivalent inductance of the transformer at the turbine end of each wind turbine unit is the same. $L_T$ is the equivalent inductance of the wind farm step-up transformer; $L_g$ is the equivalent inductance of the line, $L_{sys}$ is the system equivalent inductance, $C_{up}$ is the grid connection capacitance, $N_j$ is the serial number of the fan at the end of branch j, and $N_0 = 0$ is recorded. Defining Short Circuit Ratio (SCR):

$$SCR = \frac{U_{sys}^2}{\omega_\phi S_w (L_g + L_{sys})}$$ (1)

where $U_{sys}$ is the effective value of system voltage, $\omega_\phi$ is the grid angular frequency, and $S_w$ is the installed capacity of the entire wind farm.

2.2 Linearization model of PMSG unit

The topology of PMSG is shown in Figure 2, where the subscripts abc and dq represent the coordinates of abc and dq, respectively. $i_{abc}$ is the terminal current, $u_{abc}$ is the terminal voltage, $e_{abc}$ is the grid-connected voltage, $\theta_{abc}$ is the grid-connected current.

The permanent magnet wind power generation system consists of a wind turbine, a permanent magnet
current reference value \( i_{\text{sdref}} \) through the PI link. The difference between the stator dq axis current \( i_{\text{sdq}} \) and its respective reference values are used to form a reference value for the stator port voltage \( u_{\text{sdq}} \) through the PI link.

The grid side converter adopts grid-connected voltage vector control, with a dq transformation reference angle provided by the phase-locked angle \( \theta_{\text{PLL}} \), and its terminal voltage \( u_d \) determined by the external circuit. The error between the voltage of DC capacitor \( u_{\text{dc}} \) and its reference value \( u_{\text{dcref}} \) is output as the grid side d-axis current reference value through the PI link. The grid side converter adopts grid-connected voltage vector control, with a dq transformation reference angle provided by the phase-locked angle \( \theta_{\text{PLL}} \), and its terminal voltage \( u_d \) determined by the external circuit. The error between the voltage of DC capacitor \( u_{\text{dc}} \) and its reference value \( u_{\text{dcref}} \) is output as the grid side d-axis current reference value \( i_{\text{dref}} \) through the PI link. The q-axis current reference value of \( i_{\text{qref}} \) is set to 0.

The dq axis current \( i_{\text{dq}} \) of the grid side filtering inductor is subtracted from its respective reference values and then passed through the PI link to form a reference value \( e_{\text{dq}} \) for the internal electromotive force of the inverter. The output voltage reference values of the machine side and grid side inverters are modulated by PWM and output pulse signals to control the inverters on each side.

Linearize the PMSG model at a certain operating point to obtain its state space as shown in equation (2):

\[
\begin{align*}
\frac{d}{dt} \Delta x_k &= A_k \Delta x_k + B_k \Delta u_k \\
\Delta y_k &= C_k \Delta x_k + D_k \Delta u_k
\end{align*}
\]

In equation (2), the subscript \( k \) represents the serial number of the fan. \( \Delta u_k = [\Delta u_{\text{abc}}, \Delta u_{\text{dc}}] \) is the incremental value of the input variable, which refers to the component of the PMSG terminal voltage disturbance in the common \( xy \) coordinate system. \( \Delta y_k = [\Delta e_{\text{abc}}, \Delta e_{\text{dc}}] \) is the incremental output variable, which refers to the x-axis and y-axis components of the potential disturbance inside the grid side converter. \( \Delta x_k \) is a micro increment of the state variable \( x_k \), which is a vector space composed of real vectors.

### 2.3 Collecting network model

\[
\begin{align*}
\begin{aligned}
\left\{ \begin{array}{l}
\dot{i}_{\text{gxy}} = \dot{i}_{\text{gxy}} - i_{\text{g}(k+1)\text{xy}} & N_j-1 < k < N_j \\
\hat{i}_{\text{gxy}} = \hat{i}_{\text{gxy}} \\
u_{\text{kxy}} &= u_{\text{kxy}} + L_T \sum_{l=0}^{M-1} \left[ \dot{i}_{\text{k}(N_l+1)\text{xy}} - \Omega_{\text{k}} \hat{i}_{\text{k}(N_l+1)\text{xy}} \right] \\
&+ \sum_{h=N_j+1}^{N_j} L_{\text{ch}} \left[ \dot{i}_{\text{habc}} - \Omega_{\text{k}} \hat{i}_{\text{habc}} \right] \\
&+ L_{\text{f}} \left[ \dot{i}_{\text{gxy}} - \Omega_{\text{k}} \hat{i}_{\text{gxy}} \right]
\end{array} \right.
\end{aligned}
\end{align*}
\]

The synchronous generator, a four-quadrant inverter, and a controller. The power generation system adopts maximum power point tracking and outputs the rotor speed reference value \( \omega_{\text{ref}} \) based on the wind speed \( v_w \).

The machine-side inverter adopts rotor flux orientation control, the rotor angle \( \theta_r \) provides a reference angle for the dq transformation. The error between the rotor speed \( \omega_r \) and its reference value \( \omega_{\text{ref}} \) is output as the stator d-axis.
In the formula, \( \Omega_y \) is the grid angular frequency. Based on equation (4), \( h_{cxy} \) is the state variable, \( u_{cxy} \) and \( o_{cxy} \) are inputs, and \( u_{kxy} \) and \( i_{kxy} \) are outputs, establishing the state space of the collector network as follows:

\[
\begin{aligned}
\frac{d}{dt} \Delta x_c & = A_c \Delta x_c + [ B_{c1} \quad B_{c2} ] [ \Delta u_{c1} \quad \Delta u_{c2} ]^T \\
\Delta y_c & = C_c \Delta x_c + [ D_{c1} \quad D_{c2} ] [ \Delta u_{c1} \quad \Delta u_{c2} ]^T
\end{aligned}
\] (5)

In equation (5), \( \Delta u_{c1} \) and \( \Delta u_{c2} \) are the input variables, \( \Delta y_{k1} \) and \( \Delta y_{k2} \) are the output variables, \( \Delta x_{c} \) is the state variable; \( A_c \in \mathbb{R}^{2N \times 2N} \), \( B_{c1} \in \mathbb{R}^{2N \times 2N} \), \( B_{c2} \in \mathbb{R}^{2N \times 2N} \), \( C_c \in \mathbb{R}^{4N \times 2N} \), \( D_{c1} \in \mathbb{R}^{4N \times 2N} \), and \( D_{c2} \in \mathbb{R}^{4N \times 2N} \) are the state space matrices of the collector network.

\[
\begin{aligned}
\Delta x_c & = [ \Delta i_{1xy}^T \cdots \Delta i_{Nxy}^T ]^T \\
\Delta u_{c1} & = [ \Delta e_{1xy}^T \cdots \Delta e_{Nxy}^T ]^T \\
\Delta u_{c2} & = \Delta u_{cxy} \\
\Delta y_c & = [ \Delta i_{1xy}^T \Delta u_{1xy}^T \cdots \Delta i_{Nxy}^T \Delta u_{Nxy}^T ]^T
\end{aligned}
\] (6)

2.4 Linearization model for wind farms

Rewrite equations (2)–(7) in the following form:

\[
\begin{aligned}
\frac{d}{dt} \Delta x_{rk} & = A_{rk} \Delta x_{rk} + B_{rk} \Delta u_{tkxy} \\
\Delta e_{kxy} & = [ C_{rk1} \quad C_{rk2} ] [ \Delta x_{rk} \quad \Delta i_{kxy} ]^T
\end{aligned}
\] (7)

In the formula, \( \Delta x_{rk} \) is the vector composed of 18 other state variables in \( \Delta x_k \) except for \( i_{kxy} \). And \( C_{rk2} \) is a 0 matrix with 2 rows and 2 columns. Considering that the current \( i_{kxy} \) of the wind turbine is a linear combination of the current \( i_{kxy} \) of the collector network. Abbreviate equation (7) as

\[
\begin{aligned}
\frac{d}{dt} \Delta x_{rk} & = A_{rk} \Delta x_{rk} + B_{rk} [ \Delta i_{kxy} \quad \Delta u_{tkxy} ]^T \\
\Delta e_{kxy} & = C_r \Delta x_{rk}
\end{aligned}
\] (8)

In equation (7):

\[
\begin{aligned}
A_{rk} & = A_{k11} \\
B_{rk} & = [ A_{k12} \quad B_k ] \\
C_r & = C_{k1}
\end{aligned}
\] (9)

Integrate equations (5)–(9) establishing the state space of the entire wind farm:

\[
\begin{aligned}
\frac{d}{dt} \Delta x_F & = A_F \Delta x_F + B_F \Delta u_F \\
\Delta y_F & = C_F \Delta x_F + D_F \Delta u_F
\end{aligned}
\] (10)

In equation (10), \( \Delta u_F \) is the input variable, \( \Delta y_F \) is the output variable, \( \Delta x_F \) is the state variable; \( A_F \in \mathbb{R}^{20N \times 20N} \), \( B_F \in \mathbb{R}^{20N \times 20N} \), \( C_F \in \mathbb{R}^{2 \times 20N} \), \( D_F \in \mathbb{R}^{2 \times 2} \).

See the equation (11) top of the next page:

\[
\begin{aligned}
\Delta x_F & = [ x_{r1}^T \cdots x_{rn}^T \quad x_{c}^T ]^T \\
\Delta u_F & = u_{cxy} \\
\Delta y_F & = i_{kxy}
\end{aligned}
\] (12)

2.5 Linearization model of the whole system

The power grid subsystem consists of \( L_g \), \( L_{sys} \), and \( C_{cp} \), and its dynamic behavior is described by equation (13):

\[
\begin{aligned}
\frac{d}{dt} \Delta x_g & = A_g \Delta x_g + B_g \Delta u_g \\
\Delta y_g & = C_g \Delta x_g + D_g \Delta u_g
\end{aligned}
\] (13)

Equation (13) has a total of 4 orders, where \( \Delta u_g \) is the input variable, which refers to the sum of current disturbances at the wind farm grid connection points. \( \Delta y_g \) is the output...
variable of the power grid and the voltage disturbance of 
\( C_{cp} \). \( \Delta x_g \) is the grid state vector composed of \( C_{cp} \) xy axis voltage and AC grid reactance xy axis current increment. 
\( A_g \in \mathbb{R}^{4 \times 4}, B_g \in \mathbb{R}^{4 \times 2}, C_g \in \mathbb{R}^{2 \times 4} \) and \( D_g \in \mathbb{R}^{2 \times 2} \) are state space matrixes of the power grid.

Combining equations (11)–(13), obtain the state space of the entire PMSG wind farm interconnection system at order 20N+4:

\[
\begin{align*}
\frac{d}{dt} \Delta x_{sys} &= A_{sys} \Delta x_{sys} \\
\Delta x_{sys} &= \begin{bmatrix} \Delta x_{g}^T \\ \Delta x_{p}^T \end{bmatrix}^T \\
A_{sys} &= \begin{bmatrix} A_g & B_g C_p \\ B_g C_F & A_p \end{bmatrix}
\end{align*}
\]

The formula, \( \Delta x_{sys} \) represents the increment of the entire system state variable. \( A_{sys} \in \mathbb{R}^{(20N+4) \times (20N+4)} \) is the overall system state matrix.

3 Order reduction calculation of SSO mode for PMSG wind farm interconnection

3.1 Principle of model reduction

The general idea of model reduction is shown as follows [30]:

Given an n-order linear time-invariant system with input and output

\[
\begin{align*}
\frac{d}{dt} x &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

\( A, B, C, \) and \( D \) are the state space matrixes. \( x \) is its state variable, \( u \) and \( y \) are its input and output variables, respectively. Find the reduced transformation matrix \( W, V \in \mathbb{R}^{n \times r} \). Make the system:

\[
\begin{align*}
\frac{d}{dt} \tilde{x} &= \tilde{A} \tilde{x} + \tilde{B} \tilde{u} \\
\tilde{y} &= \tilde{C} \tilde{x} + \tilde{D} \tilde{u}
\end{align*}
\]

\( \tilde{A} = W^T A V \in \mathbb{R}^{r \times r}, \tilde{B} = W^T B \in \mathbb{R}^{r \times p}, \tilde{C} = C V \in \mathbb{R}^{p \times r}, \)

\( D = D \) are the state space matrixes of the reduced order system. \( R \) is the target order and \( r < n \). \( \tilde{x} \) are state variables for reduced order systems. \( \tilde{y} \) are output variables for reduced order systems. If matrix \( V \) is a standard column orthogonal matrix, then \( W = V \). For the reduced order system of equation (17), it is required that the dynamic characteristics of its dominant characteristic frequency band are close to the original system.

The central idea of the SSO mode reduction calculation for PMSG wind farm interconnection is to find the reduction transformation matrixes which are \( W \) and \( V \), ensuring the error between the SSO modes of the full order matrix \( A_{sys} \) and its reduction matrix \( A_{sys} \) is small enough. In practical application, in order to match the convergence of specific algorithms, corresponding preprocessing is often performed on \( A_{sys} \) before order reduction.

3.2 Arnoldi method

The Arnoldi method was developed by American scholar W. E. Arnoldi proposed in reference [31], which can be used to approximately solve large-scale subsets of eigenvalues. The following text will introduce the basic principle of this method and modify it to be applicable to the SSO problem of PMSG wind farm interconnection [32–35].

3.2.1 Arnoldi method basic steps

The central idea of the Arnoldi method is to construct a set of standard column orthogonal vectors \( v_1, v_2, \ldots, v_r \) in a given Krylov subspace, and use \( V = [v_1, v_2, \ldots, v_r] \) and \( V^H \) as the reduction transformation matrix. The specific steps for order reduction are as follows:
1. For the n-th order objective matrix $A$, given any $n$-dimensional non-zero column vector $b$, the first column base $v_1$ in the reduced order transformation matrix $V$ is obtained by unitizing it.

2. Calculate the number $a+1$ column of standard orthogonal vectors.

$$a = \frac{\text{rank}(A)}{C_0}$$

3. Organize the reduced order transformation matrix $V = [v_1, v_2, \ldots, v_r]$.

4. The elements of reduced order system state space matrix $\hat{A}$ are as follows:

$$\hat{A}_{\beta, x} = v_{\beta}^T A v_x$$

$$\hat{A}_{\beta, x} = \begin{cases} A v_x - \sum_{\xi=1}^{r} <A v_{\xi}, v_{\xi}> v_{\xi} \quad &x = \beta - 1 \quad (19) \\ \hat{A}_{\beta, x} = 0 \quad &x < \beta - 1 \end{cases}$$

$\lambda$ and $x_j$ represent the eigenvalues and corresponding eigenvectors of matrix $A$, respectively. $I$ represents the identity matrix. For characteristic equations $(A - \lambda I)$ $x_j = 0$, the approximate value of $x_j$ can be obtained in Krylov subspace span{ $v_1, v_2, \ldots, v_r$ }, which is composed of $v_1, v_2, \ldots, v_r$. If $x = V y_\pi$, then $(I^{r+1} A) y_\pi - \lambda_\pi y_\pi = 0$. Obviously, $y_\pi$ is the reduced matrix $A$ corresponding to its characteristic eigenvector of characteristic value $\lambda$. By solving $\lambda_\pi$, then an approximate subset of the eigenvalues of the full-order system can be obtained. As the order $r$ increases, $\lambda_\pi$ will gradually converge to the eigenvalues with the maximum and minimum modulus values in the original system [27].

### 3.2.2 Re-orthogonal process

The Arnoldi method is theoretically correct, but in numerical calculations, the subsequently generated vectors will lose orthogonality with each other due to the accumulation of errors during iteration, causing the algorithm failure. Therefore, it is necessary to perform re-orthogonalization on the vector generated by equation (18) to maintain the orthogonality with the previous vector [32–35]. Perform improved Gram-Schmidt orthogonalization on the vectors generated in equation (18) [33]:
Table 1. Parameters of PMSG.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power of wind turbines $P_N$</td>
<td>MW</td>
<td>1.5</td>
</tr>
<tr>
<td>Rated frequency of permanent magnet machine $f_{1N}$</td>
<td>Hz</td>
<td>30</td>
</tr>
<tr>
<td>Rated voltage of permanent magnet machine $U_{mN}$</td>
<td>kV</td>
<td>0.72</td>
</tr>
<tr>
<td>Rotor flux linkage $\psi_f$</td>
<td>Wb</td>
<td>3.1</td>
</tr>
<tr>
<td>D-axis inductance of stator $L_{sd}$</td>
<td>mH</td>
<td>1.832</td>
</tr>
<tr>
<td>Q-axis inductance of stator $L_{sq}$</td>
<td>mH</td>
<td>0.916</td>
</tr>
<tr>
<td>Inertia time constant of rotor $T_j$</td>
<td>s</td>
<td>2</td>
</tr>
<tr>
<td>Speed outer ring $(k_{p1}, k_{i1})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine side inverter d-axis current inner loop $(k_{p2}, k_{i2})$</td>
<td>/</td>
<td>(5, 10)</td>
</tr>
<tr>
<td>Machine side inverter q-axis current inner loop $(k_{p3}, k_{i3})$</td>
<td>/</td>
<td>(5, 1000)</td>
</tr>
<tr>
<td>Rated voltage of grid side inverter $U_N$</td>
<td>kV</td>
<td>0.62</td>
</tr>
<tr>
<td>DC capacitor voltage $u_{dc}$</td>
<td>kV</td>
<td>1.15</td>
</tr>
<tr>
<td>DC capacitor $C$</td>
<td>F</td>
<td>0.09</td>
</tr>
<tr>
<td>Filter inductance of grid side inverter $L_f$</td>
<td>mH</td>
<td>0.15</td>
</tr>
<tr>
<td>DC voltage outer loop $(k_{p4}, k_{i4})$</td>
<td>/</td>
<td>(10, 1000)</td>
</tr>
<tr>
<td>D-axis current inner loop of grid side converter $(k_{p5}, k_{i5})$</td>
<td>/</td>
<td>(0.1, 50)</td>
</tr>
<tr>
<td>Q-axis current inner loop of grid side converter $(k_{p6}, k_{i6})$</td>
<td>/</td>
<td>(0.9, 50)</td>
</tr>
<tr>
<td>Filter time constant $T_s$</td>
<td>s</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  b^{(\mu+1)}_x & = (I_x - V_x V_x^T) b^{(\mu)}_x, \quad \mu = 1, 2, \ldots \\
  b^{(1)}_x & = A v_x \\
  V_x & = [v_1, v_2, \ldots, v_x] 
\end{align*}
\]  

(20)

In the formula, $\mu$ is the loop variable. This cycle continues until the ratio of modulus values before and after the operation of $b_x$ is less than a given threshold (with a value greater than 1) and ends. Afterwards, standardizing $b_x$, setting as $v_{x+1}$ to proceed to steps 2) to 4). Meanwhile, reference [27] points out that in most cases, re-orthogonalization can be effectively achieved by cycling once as shown in equation (20).

### 3.2.3 Order reduction calculation for SSO mode in PMSG wind farm interconnection

For the PMSG wind farm connected SSO mode, when using the Arnoldi method for order reduction calculation, the value of the reduction order $r$ directly affects the accuracy of the SSO mode and the speed of the calculation process. However, the Arnoldi method does not provide a method for determining $r$. Therefore, the following text discusses how to determine the value of $r$ and proposes a specific method for applying the Arnoldi method to reduce the order of PMSG wind farm connected SSO mode.

According to the properties of the Arnoldi method in Section 3.2.1, as the value of $r$ increases, the SSO mode of the order reduction matrix $A_{sys}$ will approximate the SSO mode of the full order matrix $A_{sys}$. Therefore, this article proposes an iterative method for determining the value of $r$, as follows:

1. Set an initial reduction order $r_0$ and calculate the eigenvalues for each order as the step size gradually increases in order of $\Delta r$.

![Figure 6. Calculation results of 100-PMSGs-system (stable).](image)
2. The geometric distance $d$ between the SSO mode of the current order system and the SSO mode of the previous order system in the complex plane is used as the accuracy evaluation index, and this index is iteratively calculated.

3. If $d$ is less than $10^{-3}$ in a certain iteration, it is considered to meet the accuracy, and the iteration will be stopped. The current $r$ value is the target value, and the SSO mode at this time is the calculated result.

The above iterative method ensures the accuracy of the calculation results, but it is difficult to ensure the speed of the convergence process. According to Section 3.2.1, it is known that the eigenvalues of the reduced state matrix obtained through the Arnoldi method will first converge to the eigenvalues with the maximum and minimum modulus values in the original system, as the order $r$ increases. For a PMSG wind farm connected system, the modulus of the SSO mode is much greater than that of the low-frequency mechanical mode and much smaller than that of the high-frequency electromagnetic oscillation mode with faster attenuation. Therefore, in order to obtain an SSO pattern with high accuracy, the number of iteration times will increase and the calculation speed will decrease accordingly.

To ensure the speed of the calculation process, before applying the Arnoldi method, perform the following displacement transformation on the original state matrix $A_{\text{sys}}$:

$$A'_{\text{sys}} = A_{\text{sys}} - \lambda_s I_{\text{sys}}$$  \hspace{1cm} (21)

In the formula, $A'_{\text{sys}}$ is the transformed state matrix, which is a complex square matrix. $\lambda_s$ is a complex number near the SSO mode, and $I_{\text{sys}}$ is a unit matrix of the same order as $A_{\text{sys}}$. According to equation (21), it can be seen that the eigenvalues $\lambda'_{\text{sys}}$ of matrix $A'_{\text{sys}}$ and the eigenvalues $\lambda_{\text{sys}}$ of matrix $A_{\text{sys}}$, the following relationship exists:

$$\lambda'_{\text{sys}} = \lambda_{\text{sys}} - \lambda_s$$  \hspace{1cm} (22)

Setting $\lambda_s = \omega_s + jo_s$. If the eigenvalues of $A_{\text{sys}}$, corresponding to the SSO mode of the full order system are $\lambda_{\text{ssio}}$, then $\lambda_{\text{ssio}}$ can be seen as the full order SSO mode moving left in the complex plane $\omega_s$ moving down $\omega_s$. At this point, the SSO mode of $A_{\text{sys}}$ becomes the eigenvalues with the minimum modulus, and the overall convergence process has directionality, improving the calculation speed.

After transforming $A_{\text{sys}}$ into equation (21), determine $r$ according to the iterative steps mentioned earlier, and obtain the SSO mode. But at this point, $d$ is no longer the geometric distance between SSO patterns before and after each iteration, but the distance between the eigenvalues with the smallest modulus.

The initial process is as follows:

1. Calculate the average wind speed within the field and establish a single machine equivalent model;
2. Calculate the working points of the single machine equivalent model and unfold the state space $A_s$ of the single machine equivalent model $A_s$;
3. Calculate the eigenvalues of $A_s$ and obtain its SSO mode $\lambda_{\text{ssio}}$;
4. Determine the stable operating points of each unit based on the trend;
5. Expand the full-order model at the working point to obtain the full-order state matrix $A_{\text{sys}}$.

![Figure 7. Eigenvalue distribution of 100-PMSGs-system (unstable, area 4).](image)

![Table 2. Iterative process (100 PMSGs, stable).](table)

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Current order $\lambda_{\text{ssio}} + \lambda_{\text{rm}}$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>33.2753</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>$-1.4444 + j263.3426$</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>$-1.6937 + j238.0580$</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
<td>$-1.9969 + j238.2816$</td>
</tr>
<tr>
<td>5</td>
<td>104</td>
<td>$-1.9963 + j238.2859$</td>
</tr>
<tr>
<td>51</td>
<td>124</td>
<td>$-1.9964 + j238.2858$</td>
</tr>
</tbody>
</table>
Now determine value of the parameters $k_s$, $r_0$ and $D_r$.

1. The value of $k_s$ can be selected from the SSO mode in a single machine equivalent network system formed by using the average wind speed. This value can roughly locate the SSO mode of the detailed model. And the order of the single machine equivalent model is only 24 orders, compared to detailed models with thousands of orders, the additional computational resources consumed can be almost ignored. In addition, in order to further reduce the complexity of operations, the influence of the collector network is ignored when forming a single machine model.

2. For $r_0$ and $D_r$, considering that the order of the wind farm interconnection system model using single machine equivalence is 24 orders, and the model of a single PMSG unit is 20 orders, $r_0$ and $D_r$ can be taken as 24 and 20 respectively. In this way, increasing the order of each iteration is equivalent to increasing the equivalent number of fans.

In summary, the flowchart is shown in Figure 4.

### 4 Example verification

Under the Windows 10 Professional 64-bit operating system, with Intel Core i7-5500 CPU(2.40 GHz). Based on the MATLAB software platform, a wind farm interconnection calculation system is constructed, and the SSO mode reduction calculation method based on the Arnoldi method proposed is used to reduce the SSO mode of the wind farm interconnection system. By changing factors such as system stability, collector network structure, and wind farm interconnection scale, the accuracy, applicability, and computational speed of the proposed calculation method are verified.

#### 4.1 Example 1: Validation of method effectiveness in stable/unstable states

**4.1.1 Eigenvalue distribution of full order systems**

Make the number of PMSG wind farms connected to the grid $N = 100$, and the number of convergence branches $M = 10$, each of branches include 10 units (as shown in Fig. 5), the unit parameters are shown in Table 1. To ensure that the complexity of the entire system state space matrix can be maximized within the wind speed range where SSO occurs frequently, and to maximize the effectiveness of the two reduction methods, this article selects the unit wind speed and the length of the collector line according to the following principles:

1. Randomly generate 100 different wind speeds in steps of 0.01 m/s between 4 and 8 m/s;
2. Randomly generate 100 collection line lengths in steps of 1 m between 400 and 800 m.

The current full-order system has a total of 2004 orders, and the distribution of all eigenvalues is shown in Figure 6(a); Figure 6(b) is a local magnification of Figure 6(a). At this time, the SSO mode is $-1.9964 + j238.5758$ and the system is stable. Regions 1–4 in Figure 6(a) represent four regions with concentrated distribution of eigenvalues.

Region 1: Areas with real parts less than −3000;
Region 2: The region where the real part ranges from −1000 to −500;

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Current order</th>
<th>$\lambda_s + \lambda_{rm}$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>28.3139</td>
<td>218.8732</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>4.3410 + j217.5564</td>
<td>19.9837</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>4.2738 + j237.5400</td>
<td>0.2490</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
<td>4.0280 + j237.5799</td>
<td>0.0015</td>
</tr>
<tr>
<td>5</td>
<td>104</td>
<td>4.0280 + j237.5814</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

Figure 8. Comparison of calculation results of full order system and reduced order system (100 PMSGs, stable, area 4).

Table 3. Iterative process (100 PMSGs, unstable).
Region 3: The region with real parts ranging from −500 to −100; Region 4: Areas with real parts greater than −100. For the sake of discussion, the following text will focus on the behavior of eigenvalues within region 4. Reducing $\kappa_p$ to 0.09 while keeping other parameters unchanged to obtain the distribution map of eigenvalues in region 4 as shown in Figure 7. The SSO mode in Figure 7 is $4.0281 + j237.5815$, indicating that the system is unstable.

### 4.1.2 Reduced order solving SSO mode

Firstly, solving the SSO mode in a steady state descending order. The SSO mode of the single machine equivalent model is $-1.6874 + j239.9005$, and the iteration process is shown in Table 2; The comparison of eigenvalues in the full order and reduced order eigenvalue regions 4 is shown in Figure 8.

According to Table 2, after 5 iterations, the SSO mode converges, and $A_{sys}$ is of the 124th order. From Figure 8, it can be seen that the SSO mode is well maintained before and after order reduction.

Similarly, the reduction calculation process and results in unstable states can be obtained. At this time, the SSO mode of the single machine equivalent model is $4.3897 + j239.1500$, and the iteration process is shown in Table 3. The comparison of eigenvalues in region 4 of the full order and reduced order systems is shown in Figure 9. According to the calculation results, the SSO mode that meets the accuracy requirements was still obtained at $r = 124$ in the unstable state. The results demonstrate the effectiveness of the proposed method in stable and unstable states.

### 4.2 Example 2: Validation of method applicability under different collecting network structures

Section 4.1 has provided a topology structure for the collector network, which is referred to as Structure 1 in this article; This section will further set up the following two topological structures to verify the applicability of the method for wind farms with different structures:

**Structure 2:** Make the number of PMSG wind farms connected to the grid $N = 100$, and the number of convergence branches $M = 5$, each of which includes 20 units;

**Structure 3:** Ensure that the number of PMSG wind farms is connected to the grid is $N = 100$, and each unit

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**Figure 9.** Comparison of calculation results of full order system and reduced order system (100 PMSGs, unstable, area 4).

**Figure 10.** Circuit topology of collector.
is connected in parallel to the generator end busbar through a collection line.

Figure 10 shows the internal wiring diagrams of two types of wind farms with different structures; The operating conditions of the unit and the length of the collector line remain the same as in Section 4.1.

Due to the increasing impedance between the fan near the end of the branch and the grid connection point in structure 2, its SSO mode tends to be more unstable compared with structure 1, and the oscillation frequency decreases by 0.7634 Hz to $-1.7153 + j233.7053$. The distribution of eigenvalues in region 4 before and after the descending order of this structure is shown in Figure 11. As shown in Figure 11, the reduced order SSO mode has good computational accuracy.

The SSO mode of Structure 3 is $-2.1014 + j239.9501$, which is more stable than Structure 1 and Structure 2. The distribution of characteristic values in region 4 is shown in Figure 12; The reduced order and full order SSO modes in Figure 12 are basically consistent. In summary, the applicability of the method proposed in this article to wind farms with different topology structures has been verified.

4.3 Example 3: Rapid validation of methods under different wind farm scales

In this section, under the condition of the chain collector network structure, unit and network parameters as shown in Table 1, $N$ is gradually expanded from 100 units to 600 units with a step size of 100 units, and each branch is maintained to contain 10 units; Comparing the computation time of full order and reduced order to verify the speed of the proposed method.

Correspondingly, generating the wind speed of the unit and the length of the collector line according to the method in Section 4.1. This method can generate a total of 400 different wind speeds; Therefore, when the number of fans is less than 400, the wind speed of each unit should be taken within the above range and ensured to be different from

Table 4. Reduced order and computation time under different wind farm scales.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Full order</th>
<th>Full order computation time (s)</th>
<th>Iterations</th>
<th>Reduced matrix order</th>
<th>Reduced computation time (s)</th>
<th>Order reduction time/full order time consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 units</td>
<td>2004</td>
<td>5.45</td>
<td>5</td>
<td>124</td>
<td>3.28</td>
<td>60.18%</td>
</tr>
<tr>
<td>200 units</td>
<td>4004</td>
<td>23.87</td>
<td>5</td>
<td>124</td>
<td>9.83</td>
<td>41.18%</td>
</tr>
<tr>
<td>300 units</td>
<td>6004</td>
<td>86.02</td>
<td>5</td>
<td>124</td>
<td>20.32</td>
<td>35.24%</td>
</tr>
<tr>
<td>400 units</td>
<td>8004</td>
<td>193.11</td>
<td>5</td>
<td>124</td>
<td>34.13</td>
<td>17.67%</td>
</tr>
<tr>
<td>500 units</td>
<td>10004</td>
<td>352.75</td>
<td>5</td>
<td>124</td>
<td>52.70</td>
<td>14.94%</td>
</tr>
<tr>
<td>600 units</td>
<td>12004</td>
<td>594.77</td>
<td>5</td>
<td>124</td>
<td>75.89</td>
<td>12.76%</td>
</tr>
</tbody>
</table>

each other. When the number of fans is equal, or greater than 400, these 400 wind speeds will be allocated to 400 units, and the wind speeds of other units will be randomly generated in steps of 0.01 m/s between 4 and 8 m/s. This principle also applies to the generation of collector line length.

The time required for full order and reduced order calculations is shown in Table 4. According to Table 4, as the scale of the wind farm expands from 100 units to 600 units, the proportion of reduced order calculation time relative to full order time monotonically decreases from 60.18% to 12.76%. This result indicates that the order reduction method proposed in this article effectively saves computational costs; the larger the scale of the wind farm, the more obvious the speed of the reduction calculation method.

In Table 4, the calculation time unit for the full order system is seconds, which is acceptable for general analysis. However, the above results only represent the computation time in the MATLAB computing environment and do not mean that there will be the same computation speed in other environments. Nevertheless, the quantitative relationship between the time consumption of full order and reduced order calculations in Table 4 has certain reference significance for practical engineering calculations and the development of new functions in other power engineering calculation software in the future.

5 Conclusion

Based on the full-order wind farm networking model, SSO will face the problem of “dimensionality disaster”. This article proposes a reduced order calculation method for PMSG wind farm connected in SSO mode based on the Arnoldi method, which provides a fast and effective evaluation method for wind farm connected in SSO analysis. The main conclusions are as follows:

1. The SSO mode reduction calculation method proposed for wind farm networking systems can take into account the influence of factors such as the number of wind turbines, the collection network, and unit operating conditions, and has strong applicability.

2. The calculation results of the stable/unstable scenarios of SSO connected to 100 wind farms and the topology structure of series/parallel collector networks show that the geometric distance between the feature roots of the SSO mode calculated by the proposed reduction method and the full order model SSO mode is less than $10^{-4}$, indicating the accuracy of the proposed reduction calculation method. The scale of the wind farm has been expanded from 100 units to 600 units. Compared with the full order calculation time, the reduced order calculation time has been reduced from 60.18% to 12.76%, indicating that the larger the wind farm scale, the more obvious the speed of the proposed reduced order calculation method.

References

1 Erdiwansyah, Mahidin, Husin H., et al. (2021) A critical review of the integration of renewable energy sources with various technologies, Prot. Control Mod. Power Syst. 6, 1, 3.