Science and Technology for Energy Transition **78**, 2 (2023)

https://doi.org/10.2516/stet/2022023

Available online at:
stet-review.org

REGULAR ARTICLE OPEN ACCESS

NACA2412 airfoil based method for design and aerodynamic analysis of small HAWT using modified BEM approach

Devashish Jha[1,*] and Saket Saurabh[2]

[1] Madanapalle Institute of Technology & Science, Post-Angallu, Madanapalle 517325, Andhra Pradesh, India
[2] Katihar Engineering College, Katihar, Bihar 854109, India

Received: 7 August 2022 / Accepted: 24 November 2022

Abstract. Efficient utilization of wind energy depends on careful selection and suitable design of Wind Turbine (WT) blades. Small scale Wind Turbines (SWT) normally operate in the low range of *Angle of Attack* (*AoA*). This makes the task even more difficult for designing and optimization of WT blades. This article deals with an airfoil based computational approach to design the blade for a standalone small scale Horizontal Axis Wind Turbine (HAWT). A procedure has been proposed to find the important parameters and analyze the performance characteristics for a three bladed HAWT operating under the wake rotation. Computational code has been written to find optimum blade profile. Twist angle variation, chord length and other related parameters are determined with the help of program. Comparison between different types of airfoil has been made to figure out the most suitable one. Airfoil selection and design approach are intended to make Wind Turbine blade efficient specially under low range of *AoA*. Characteristics of the Wind Turbine obtained analytically from this procedure are compared with several other reported earlier in some of the literatures. The result obtained by the proposed procedure is simpler and more efficient than BEM theory – a method normally employed for blade design.

Keywords: Small Wind Turbine, Aerodynamics, Blade designing, Airfoil, Lift and drag coefficient, Chord, Twist.

Abbreviations

NACA	National Advisory Committee for Aeronautics
WT	Wind Turbine
HAWT	Horizontal Axis Wind Turbine
B	Number of blades
Re	Reynolds number
C_L, C_D	Lift and drag coefficient
F_L, F_D	Lift and drag force
λ, TSR	Tip speed ratio
i	Angle of Attack
θ_T	Twist angle
c	Chord length
θ_p	Sectional pitch angle
C_p	Power coefficient
a	Axial induction factor
a'	Angular induction factor
I	Angle of inclination/angle of relative wind
AoA	Angle of Attack

* Corresponding author: devashish.sit@gmail.com

1 Introduction

Wind as an alternative to fossil fuel, is one of the most available and exploitable form of energy. In most of the cases, it is widely used to generate electrical power by revolving Wind Turbine (WT) blades around its rotor axis [1–3]. The amount of power generated relies on judicious selection and design of Wind Turbine blades. Wind Turbine are mainly of two types (horizontal and vertical axis WT) and generally available in the range from a few Watts to several Megawatts. Nowadays, small scale standalone Horizontal Axis Wind Turbines (HAWT) are being widely employed to produce the power for various places in remote communities, including off-grid residences, telecom towers, offshore platforms, schools, hospitals and similar others where grid is not accessible. IEC 61400-2 defines the range for swept area of Small WT (SWT) to be less than 200 sq-m, corresponding to approximately less than 50 kW output power generation [4]. Large horizontal HAWT are often located in the areas where optimum wind condition is most likely to prevail. On the contrary, Small WT (SWT) are frequently subjected to intermittent wind profile which is not usually beneficial to produce desired power. Moreover, it has low inertia and large portion of it function under low

Reynolds number (*Re*) due to small diameter of rotor swept area and low wind velocity. As the result, it experiences low lift force and high drag force. In addition to this, Wind Turbine characteristics (power and torque coefficient) are largely dependent upon *Reynolds number*, which also varies with wind speed. Aerodynamic characteristics associated with low *Reynolds number* flow must be properly addressed, particularly in small WT under the low tip speed ratio at which the rotor blade may undergo flow separation and dynamic stall [17, 18]. Therefore, development of an effective WT under this condition requires careful selection of airfoil and effective design of blades, so that it could be able to successfully perform under various wind speeds even at low *Reynolds number*. Arodynamic efficiency of WT-blades can be improved by making it efficient in extracting energy from the wind. It is required to design parameters appropriately for a SWT to improve the efficiency and performance characteristics along with its associated control schemes [5].

Many research works have been proposed so far for optimal design of WT blade to meet the desired performance characteristics. Majority of them are devoted to normally existing large WT and little has been done for SWT [6, 11, 12]. One non-dimensional parameter that largely affects the performance of small WT is *Reynolds number*. Its value is very less in the case of SWT and dependent upon resultant wind velocity that is also fluctuating in nature. Thus effect of *Re* must be carefully *Re* examined while designing small WT, as value of it changes along the blade and is not fixed. In [13], an airfoil has been designed after testing several low Reynolds number airfoils using XFOIL and selecting one of them. XFOIL is an interactive program for the design and analysis of airfoil, whose data (lift and drag) cannot be much trusted for the small WT blade operating in low Angle of Attack and *Reynolds number*. In [7], methodology has been developed for the aero-structural and aero-dynamic design of SWT while taking the effect of several related parameters into consideration. In [6], a complete picture of modern state of art Wind Turbine blade design including, airfoil selection, shape/quantity, and optimum Angle of Attack in the form of review have been presented. However, in none of these papers ([6] and [7]) effect of the *Reynolds number* is considered, a significant factor that should be taken care while designing SWT. One important aspect that should be investigated while designing the SWT is its starting torque behaviour. In [8], NACA63-415 airfoil, has been taken for modelling and designing of SWT while taking cognisance of its starting torque behaviour. In [9], design and optimization of small HAWT have been done by using MATLAB programming based on Blade Element Momentum (BEM) theory. Performance was investigated with multiple airfoils taken for analysis and comparison. There is usually a compromise between starting torque and running behaviour while designing SWT. Selection between two is done on the basis of performance requirement. In [10], SG6043 airfoil has been taken for designing the 4 kW Wind Turbine. An algorithm has been developed in MATLAB to discover the parameter radius, chord length and blade twist. Based on the obtained parameter, performance analysis was carried out under changing wind speed and pitch angle. However, the paper does not introduce the effect of wake rotation which is produced by flow behind rotating generator and responsible for less energy extraction from WT. A survey of various blade profile and optimization have been explained for SWT in detail, that undergoes *Re* less than 500,000 [11]. In [15], a detailed review is presented for the design and performance investigation of WT both experimentally and numerically. Blade element-momentum theory has been employed for blade design of a Wind Turbine [11, 12, 19, 20] and impact of turbulence studied [14]. Starting torque behaviour of Wind Turbine has been invertigated in [15].

After going through aforementioned literature it can be deduced that the main goal of aerodynamic design for a WT blade is to find out its geometrical structure (chord, twist, and other related parameter) along the blade length for a specific airfoil to meet the power requirement and performance characteristics. Many works that are found in several literatures are devoted to design and analysis of large scale Wind Turbine. Small WTs, which usually operate under low *Reynolds number*, inertia and AoA, have not found much attention in the past literatures. Lift and drag data of specific airfoil is essential for design and performance investigation. These data are being usually taken from already available data sheet. The data are obtained experimentally and may not be reliable for small scale WT operating under low. Proper choice for the airfoil should be made after examining lift and drag data in addition to other system related requirement.

This article stepwise explicates design method for a 4 kW Wind Turbine with the more realistic aerodynamic model using numerical approach and program code. Lift and drag data are analysed by taking several airfoils into consideration. Choice of best one among them has been made after drawing a comparison between them. Optimal geometrical structure of blade has been tried to discover while taking wake rotation into the account. After following step by step procedure for design of blades, its performance is investigated and compared with experimentally obtained results in some of the literatures. The exposition is aimed to familiarise the reader with logical geometrical structure of SWT and hypothetical description for an optimal blade design with which various forces can be assessed and basic design made. This work may find applicability in the design of blade for Small scale Wind Turbine, which is to be installed at the isolated location far away from an electricity grid.

The article is structured as follows: Section 1 gives introductory information about the work. Section 2 provides basic idea about airfoil terminology and forces acting on a Wind Turbine blade-airfoil by virtue of which rotational motion is imparted in WT rotor. Effect of Reynolds number on lift and drag is illustrated by presenting a set of data correlation. Section 3 draws a comparison between commonly used airfoil and makes choice for best suited one as per requirement of the system. Section 4 has been divided into several sub sections. It mainly throws light on forces acting on an airfoil and design aspects of WT blades with its possible geometrical shape. Section 5 explains about design alogorithm and deals with detailed steps. Section 6 discusses about results obtained over whole paper. In the last, Section 7 concludes the whole work.

2 Small Wind Turbine technology and concept of aerodynamics

Wind flow exerts a resultant force on WT blade, which causes rotation in the rotor (Fig. 1). Experiments have shown that power generated from WT largely depends upon mean of wind speed and very less on turbulent fluctuations around it. Resultant of lift and drag force is responsible for rotation of blade about its axis. The development of an efficient WT depends upon selection and design of blades.

2.1 Airfoil terminology and operation

To design and analyse the characteristics with more accurate results, blade is divided into several small sections (airfoil). Numerous terminologies are used to describe an airfoil and its geometry. Blade of length R is divided into multiple smaller blade elements of width dr and chord length c, as shown in Figure 2.

Flow of air over airfoil causes pressure difference due to change in velocity in its convex and concave surfaces. This difference produces lift force perpendicular to the direction of wind-flow. The drag force which is towards the direction of flow exerts thrust force on the blade. Ratio of lift to drag forces should be highest throughout length of the blade to capture maximum power from wind. It is observed that lift to drag ratio for each airfoil is maximum at specific Angle of Attack. Thus, in order to maximise the power blade twist should be varied accordingly so as to maintain that specific Angle of Attack. The lift and drag forces are dependent on their respective coefficient values. The thrust and power of WT depend upon lift and drag coefficient of each sections. These coefficients, in general may be known by experimental investigations and again taken for next procedures. However, computational results obtained from those experimentally determined data cannot be trustworthy. Therefore, a data correlation sample is presented here to determine more accurate and reliable lift and drag coefficient under wide variation of AoA as suggested by Wood [1].

Maximum value of drag coefficient is given by:

$$C_{D,\max} = 1.11 + 0.018 \times AR \quad \text{for Aspect} - \text{ratio} < 50,$$
$$= 2.01 \quad \text{for Aspect} - \text{ratio} \geq 50. \tag{1}$$

Aspect-Ratio (AR) is surface area of blade divided by its total span. Expression of lift and drag coefficient is given as follows:

$$C_D = C_{D,\max} \sin^2 i + k_1 \cos i, \tag{2}$$

$$C_L = 0.5 C_{D,\max} \sin 2i + k_2 \frac{\cos^2 i}{\sin i}, \tag{3}$$

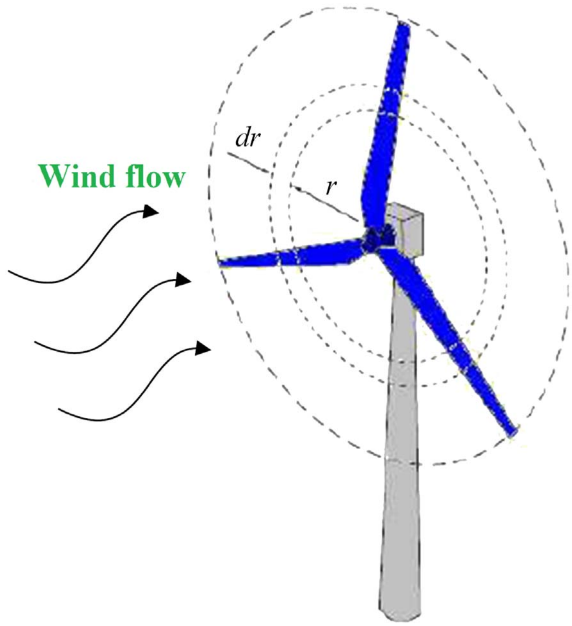


Figure 1. A HAWT having three blades.

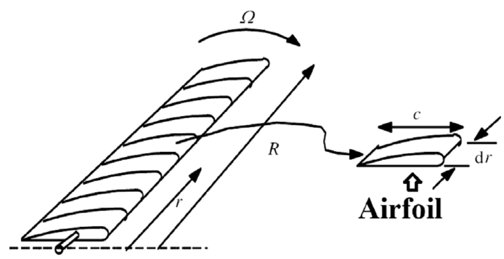


Figure 2. An airfoil out of blade.

where,

$$k_1 = (C_{Ds} - C_{D,\max} \sin^2 i_s) / \cos i_s \tag{4}$$

and,

$$k_2 = \frac{(C_{Ls} - C_{D,\max} \sin i_s . \cos i_s) \sin \alpha_s}{\cos^2 i_s}. \tag{5}$$

C_{Ds} and C_{Ls} are actually experimental value of drag and lift coefficient respectively for a specific tiny value of AoA, i_s. To order to get the relationship between lift and drag with AoA, NACA2412 airfoil is being brought into the operation under different Re. It can be observed in Figure 3, that there is significant variation of lift in the range of low AoA, whereas drag does not vary too much under lower value of Re. In view of this, special care has to be taken for blade-design of small HAWT which normally operates under low AoA and Re.

3 Choice of airfoil

Lift coefficient is meant for generating torque responsible of rotating the rotor. Variation in lift alongwith drag

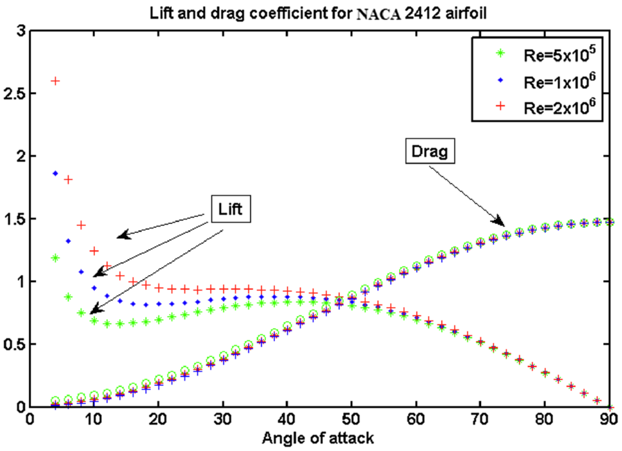


Figure 3. Lift and drag coefficient variation with *Angle of Attack* (i).

coefficient of few airfoil has been shown in Figure 3. It is also known that drag and tip losses cause the reduction in power coefficient. A relation showing dependency of maximum power coefficient upon lift, drag coefficient and number of blades has been given by equation (6)

$$C_p = 0.59\lambda\left[\lambda + \frac{1.32 + \left(\frac{\lambda - B}{20}\right)^2}{B^{2/3}}\right]^{-1} - \frac{0.59\lambda^2}{\frac{C_L}{C_D}\left(\lambda + \frac{1}{2B}\right)}. \qquad (6)$$

It can be well deduced from equation that maximum power coefficient depends upon number of blades, tip speed and ratio of lift and drag coefficient. It can be concluded that an analysis of lift to drag ratio of different airfoils is essential for selecting most suitable one as per the system requirement. Moreover, the aerodynamic of SWT is highly influenced upon resultant wind velocity striking on it, particularly under low value of Re. Thus, suitable airfoil should be chosen for optimum blade design of the blade. Discussions on several airfoils used for blade designing can be found in numerous literatures. Some of the most widely used airfoils are NACA4412 and NACA2412 from *National Advisory Committee for Aeronautics*, SG6043 from Selig–Giguere and SD7062 from Selig–Donovan group. It is known that lift force provides rotational velocity to the WT rotor while the drag force exerts a thrust force on the blade. In fact, ratio of lift and drag is required to be maximised for an efficient blade design, so that it could harness maximum energy from wind. A comparison of lift:drag for above mentioned airfoils based on method given by equations (1)–(5) in the low range of *Angle of Attack* has been shown in Figure 4a. It may be observed that NACA2412 airfoil has got the maximum value of lift:drag in this range. This method is a computation based approach. However, it depends upon pre-experimentally determined value of lift coefficient at small Angle of Attack. To strengthen the suitability of NACA2412 airfoil for this work, an experimental result of these airfoils has been been shown for low and even wider range of AoA in Figure 4b. It is clear from the figure that NACA2412 airfoil has got maximum Lift:drag in the low value of AoA (2°–8°). Hence, in this work, NACA2412 airfoil is being considered for design and analysis. A new airfoil, SD7062 has been introduced for the analysis. Though, this airfoil does not have significant lift:drag, its extra thickness provides the much needed strength to survive the centrifugal load on small WT blades. It may also be observed that lift:drag of airfoil, SD7062 may be more impressive in the very low (negative) range of AoA.

4 Blade design theory

Power generated from the Wind Turbine is dependent upon the interaction between blade of the turbine and the wind. If same airfoil is assumed throughout the blade, maximum power can be extracted if ratio of lift and drag could be maximized in all section of the blade. Blades being critical component of WT consists of several airfoils that actually interact with wind to convert the wind energy to mechanical power of Wind Turbine by rotating its rotor. Some of the geometrical parameters of WT blade that need to be taken care are blade diameter, radial distribution of chord and twist, pitch angle and tip-speed ratio. Blades sections should be designed in such a way that the lift to drag ratio is maximum throughout the blade. This optimum value is obtained at a particular angle of incidence (Angle of Attack). To maintain same Angle of Attack along the blade length, pitch angle (twist) must also change, as inclination angle varies along the entire blade length.

4.1 Design of Wind Turbine blade

In this section the geometrical structure for optimal blade shape has been determined and analyzed. Here, the analysis of WT rotor is done using Blade Element Momentum (BEM) theory, which is also called as strip theory that combines results of two popularly known theories naming momentum and blade element theory. For the design and analysis purpose, the WT-blade is supposed to be split in N-sections (elements). Representation of such blade element (airfoil) is shown in Figure 2. Each section faces certain wind velocity. Some assumptions are made to perform analysis simpler by avoiding some negligible effect.

- Each element is independent and there is no aerodynamic interaction between sections of blades. Airstream is also divided between non interacting annular stream tubes of the width same as that of blade element.
- The forces acting on blade are calculated only by lift and drag characteristics of airfoil.

For the determination of forces (thrust) produced by the WT blade, optimal blade shape needs to be figured out. For this, rotor geometrical parameter, *i.e.* variation of chord length, twist angle are to be calculated along the entire blade length.

To start with the process of obtaining the optimal blade shape, few mathematical formulation related to WT aerodynamics has to be introduced.

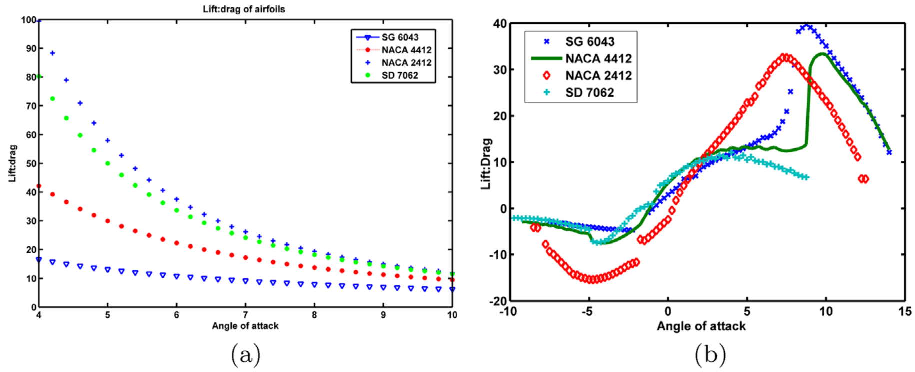


Figure 4. Lift:drag for SWT airfoil with varying (a) Theoritical; (b) Experimental approach at Re = 50,000 and Ncrit = 9.

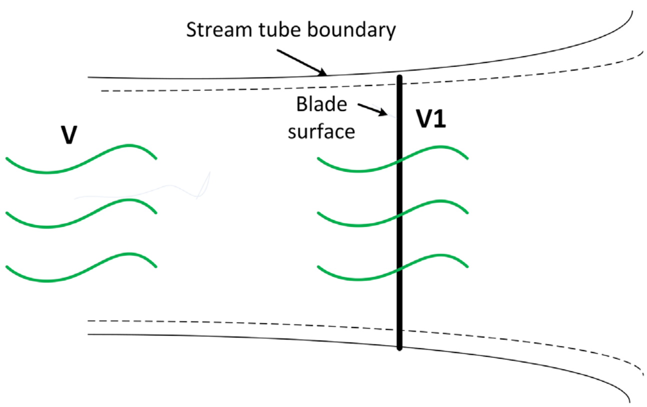


Figure 5. Wind of velocity V coming from far and passing through blade with velocity V_1.

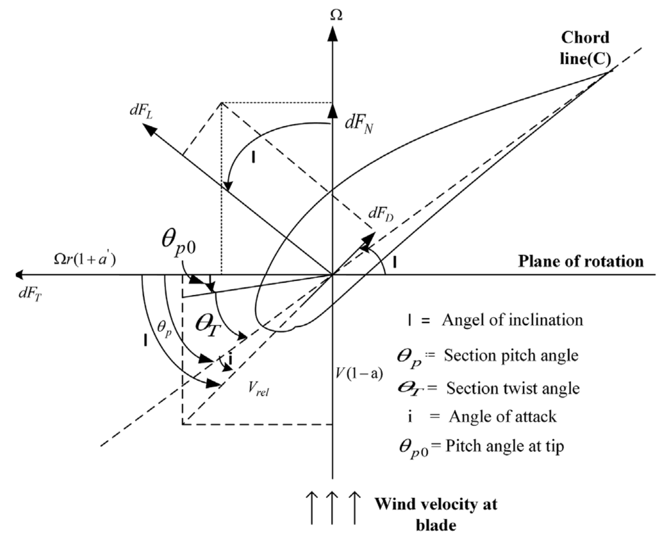


Figure 6. An airfoil of Horizontal Axis Wind Turbine showing its geometrical parameter and forces acting on it.

Fractional decrease between wind velocity far from Wind Turbine (V) and that passing through blade (V_1) is termed as axial induction factor, a (Fig. 5):

$$a = \frac{V - V_1}{V}, \tag{7}$$

where, V is the velocity of wind coming from infinity and V_1 is the velocity of wind passing through the blade. Some relation can be deduced from Figure 6:

$$\theta_T = \theta_p - \theta_{p0}, \tag{8}$$

where, θ_p is the angle between chord line and rotational plane of blade called as sectional pitch angle, θ_{p0} is the pitch angle at the tip of the blade. Angle of relative wind (inclination angle), I is expressed as, sum of *Angle of Attack*, i and sectional pitch angle, *i.e.*,

$$I = i + \theta_p. \tag{9}$$

From Figure 6, expression for "angle of inclination" may be given as;

$$\tan I = \frac{V(1-a)}{\Omega r(1+a')} = \frac{1-a}{(1+a')\lambda_r}, \tag{10}$$

where V is the speed of uninterrupted free flow (far from turbine blade) of wind, ω is the angular velocity transmitted to wind flow due to wake and $a' = \omega/2\Omega$ is termed as *Angular Induction Factor* (AIF) and ω_r is tip-speed ratio at distance r from the blade hub. It is clearly evident from the figure, that drag is along the direction of relative wind whereas lift is along its perpendicular direction.

4.1.1 Strip theory

Strip theory predicts the performance of WT by using blade element and momentum theory. These two theories can be deduced by establishing the relationship between momentum and force acting on the blade.

A blade element (airfoil) is considered for the analysis of forces acting on it. The airfoil is of thickness "dr" is placed at a distance "r" from the hub. Incremental lift and drag forces experienced by this airfoil can be expressed as:

$$\mathrm{d}F_L = 0.5 C_L \rho {V_{rel}}^2 c\mathrm{d}r, \tag{11}$$

$$\mathrm{d}F_D = 0.5C_D\rho {V_{rel}}^2 c\mathrm{d}r. \quad (12)$$

Force perpendicular to the rotational plane and that tangential to the swept circle can be obtained as:

$$\mathrm{d}F_N = \mathrm{d}F_L \cos I + \mathrm{d}F_D \sin I, \quad (13)$$

$$\mathrm{d}F_T = \mathrm{d}F_L \sin I - \mathrm{d}F_D \cos I. \quad (14)$$

With the help of (11) and (12), (13) can be expressed by:

$$\mathrm{d}F_N = 0.5B\rho {V_{rel}}^2 (C_L \cos I + C_D \sin I)c \cdot \mathrm{d}r. \quad (15)$$

Similarly, differential torque onto the blade-portion at the distance r from hub due to tangential force.

$$\mathrm{d}Q = rB\mathrm{d}F_T = 0.5B\rho {V_{rel}}^2 (C_L \sin I - C_D \cos I)cr\mathrm{d}r. \quad (16)$$

It is worth noting that drag causes increase in normal force or thrust and decrease in torque.

From the knowledge of momentum theory, torque ($\mathrm{d}Q$) and thrust ($\mathrm{d}T$) acting on a blade section are given by:

$$\mathrm{d}Q = 4a'(1-a)V\rho\pi r^3\Omega \mathrm{d}r, \quad (17)$$

$$\mathrm{d}T = 4\rho V^2 a(1-a)\pi r\mathrm{d}r. \quad (18)$$

Strip theory is basically combination of "Blade element-momentum theory" to obtain various important relationship between them and formulate method for blade design.

4.1.2 Evaluation of blade shape

For determining shape of the ideal blade some assumption are to be made. Wake rotation, drag forces and losses between blades are neglected. In this design process, ideal blade shape with known lift and drag coefficient as a function of "*Angle of Attack*", is to be designed for a certain tip-speed ratio. Angle is deliberately chosen in such a way that ratio of lift and drag is maximum throughout the blade. It allows the choices of chord and twist distribution of the blade appropriate to provide Betz limit output power. For obtaining maximum power coefficient, value of axial induction factor is taken as, $a = 1/3$ (Betz optimum rotor). Blade element theory and momentum theory have been combined (strip theory) to obtain the expression for optimum chord and twist distribution. From momentum theory, expression for differential thrust, after putting $a = 1/3$ in equation (18) is obtained as;

$$\mathrm{d}T = \frac{8}{9}\rho V^2 \pi r\mathrm{d}r. \quad (19)$$

Considering ideal case, thereby equating drag coefficient to the zero, equation (15) may be reexpressed as:

$$\mathrm{d}F_N = 0.5B\rho {V_{rel}}^2 (C_L \cos I)c \cdot \mathrm{d}r. \quad (20)$$

From Figure 6.

$$V_{rel} = \frac{V(1-a)}{\sin I} = \frac{V(1-1/3)}{\sin I} = \frac{2V}{3\sin I}. \quad (21)$$

Thrust and normal force obtained from both mentioned theories can be combined by equating equations (19), (20) and (21) to form the expression for BEM or strip theory.

$$\frac{cBC_L}{4\pi r} = \tan I \sin I. \quad (22)$$

Assuming ideal case, hence neglecting effect of wake, thus taking $a' = 0$ and $a = 1/3$ for having maximum efficiency.

5 Algorithm for Wind Turbine blade design

After obtaining derivation for blade shape considering ideal situation, design procedure is developed for a 4 kW WT blade consisting of NACA2412 type airfoil. Investigation has been done considering the power required at particular rated wind velocity.

Step 1: Determination of swept-area radius

Selection of maximum possible value of power coefficient and rated value of wind velocity to determine the radius of the Wind Turbine sweeping-area, R:

$$R = \sqrt{\frac{P \times 100}{0.5\eta\rho C_p \pi V^3}}. \quad (23)$$

An extra factor "η" is added, which is combined efficiency of various factor (*i.e.* gearbox, generator etc). For simplicity and analysis, its value is taken as unity.

Step 2: Selection of an appropriate value of TSR (λ) and blades

For the Wind Turbine application the range could be 4–10. In present work, its value has been taken to be 7. To avoid the structural and dynamic problem in SWT, number of blades less than 3 is generally not taken. Here three bladed WT is considered here.

Step 3: Determination of WT rotor performance and modification in the design of blade

Equations (15) and (16) may be expressed as a function wind velocity using equation (21) as:

$$\mathrm{d}F_N = \frac{Bc}{2\pi r}\rho\pi \frac{V^2(1-a)^2}{\sin^2 I}(C_L \cos I + C_D \sin I)r \cdot \mathrm{d}r, \quad (24)$$

$$\mathrm{d}Q = \frac{Bc}{2\pi r}\rho\pi \frac{V^2(1-a)^2}{\sin^2 I}(C_L \sin I - C_D \cos I)r^2 \cdot \mathrm{d}r. \quad (25)$$

Normally, C_D is equated to zero, as airfoil having low C_D value, put little difference on torque calculation. For the calculation of angular induction factor torque equation obtained ((17) and (25)) from BEM theory is equated to have:

$$\sigma = \frac{Bc}{2\pi r}, \tag{26}$$

where, σ is recognized as local solidity. Equating normal force (thrust) equations, (18) and (24) expression for "a" can be obtained:

$$a = \frac{(1-a)\sigma C_L \cos I}{4\sin^2 I}. \tag{27}$$

Having gone through some mathematical calculation with equations (10), (37) and (38) following vital expressions may be obtained:

$$C_L = \frac{4\sin I(\cos I - \lambda_r \sin I)}{\sigma(\sin I + \lambda_r \cos I)}, \tag{28}$$

$$a' = \frac{Bc}{2\pi r}(1-a)C_L \cdot \frac{1}{4\cos I}. \tag{29}$$

Few other relationship based upon previous equations are as follows:

$$a = \frac{a'\lambda_r}{\tan I}, \tag{30}$$

$$a = \frac{1}{1 + \frac{4\sin^2 I}{\sigma C_L \cos I}}, \tag{31}$$

$$a' = \frac{1}{\frac{4\cos I}{\sigma C_L} - 1} = \frac{1-3a}{4a-1}. \tag{32}$$

To perform the performance evaluation of Wind Turbine, power coefficient needs to be calculated, which is ratio of power developed at WT (P_{wt}) and incoming wind (P_{wind}):

$$C_p = \frac{P_{\mathrm{wt}}}{P_{\mathrm{wind}}} = \frac{\int_{r_h}^{R} \Omega dQ}{0.5\rho\pi R^2 V^3}, \tag{33}$$

where, r_h and Ω are the radius of swept area circle of a point near hub and blade-rotational velocity respectively. Replacing dQ from (25) in, (33) extended expression for C_p is obtained:

$$C_p = \frac{2}{\lambda^2}\int_{\lambda_h}^{\lambda} \frac{Bc}{2\pi r}\left(\frac{1}{\sin I}\right) C_L (1-a)^2 \left\{1 - \left(\frac{C_D}{C_L}\right)\cot I\right\} {\lambda_r}^2 \mathrm{d}\lambda_r, \tag{34}$$

$$C_p = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda} (\sin I + \lambda_r \cos I)(\cos I - \lambda_r \sin I) \sin^2 I \left\{1 - \left(\frac{C_D}{C_L}\right)\cot I\right\} {\lambda_r}^2 \mathrm{d}\lambda_r. \tag{35}$$

To have an optimal geometrical shape and obtain the performance of proposed SWT an iterative procedure has to be followed. For this, an initial value of both induction factors (a and a') needs to be determined for kth blade section, which are given as per the (42) and (43):

$$a_{k,1} = \frac{1}{1 + \frac{4\sin^2(k,1)}{\sigma_k C_{L,des} \cos I_{k,1}}}, \tag{36}$$

$$a'_{k,1} = \frac{1 - 3a_{k,1}}{4a_{k,1} - 1}. \tag{37}$$

After having knowledge of initial values of induction factors, iterative procedure starts with calculation of *Angle of inclination*, for the nth iteration:

$$\tan I_{k,n} = \frac{1}{\lambda_{r,k}}\left(\frac{1 - a_{k,n}}{1 + a'_{k,n}}\right). \tag{38}$$

5.1 Losses and suggested correction

Since, pressure in covex surface of the blade is lower than that of in concave, airflow tends to flow towards tip resulting in reduction of lift force essential for generation of torque. Hence, a correction factor as suggested by Prandtl is introduced:

$$P_{k,n} = 0.64\cos^{-1}\left[\exp\left\{\frac{0.5B\left(\frac{r_k}{R} - 1\right)}{\left(\frac{r_k}{R}\right)\sin I_{k,n}}\right\}\right]. \tag{39}$$

When blades start rotating, the air-stream behind it rotates in opposite direction causing lesser power extraction than it was without considering wake. Hence, in order to incorporate effect of wake there must be modification in geometrical structure of blade in terms of chord and twist. Blade shape taking account this effect is obtained by partially differentiating that portion of maximum power coefficient which is function of 'I' and equating it with zero. From (35):

$$\frac{\partial}{\partial I}\left\{(\sin I + \lambda_r \cos I)(\cos I - \lambda_r \sin I)\sin^2 I\right\} = 0, \tag{40}$$

$$\lambda_r = \frac{\sin I(2\cos I - 1)}{(2\cos I + 1)(1 - \cos I)}. \tag{41}$$

After few careful exploration and simplification a new geometrical shape (in form of chord and twist) can be found out

$$I = 0.66\tan^{-1}\left(\frac{1}{\lambda_r}\right), \tag{42}$$

$$c = \frac{8\pi r}{BC_L}(1 - \cos I). \tag{43}$$

Step 4: Determination of geometrical structure of blade

Entire Blade length is partitioned with N segment of equal width. Here, value of "N" is taken to be 18. Considering effect of wake rotation (a' is non zero) shape and size of kth airfoil or blade section may be computed as given below:

$$\lambda_{r,k} = \lambda(r_k / R), \tag{44}$$

$$I_k = 0.66\tan^{-1}\left(\frac{1}{\lambda_{r,k}}\right), \quad (45)$$

$$c_K = \frac{8\pi r_k}{BC_L}(1-\cos I_k), \quad (46)$$

$$\theta_{T,k} = \theta_{p,k} - \theta_{P,0}, \quad (47)$$

$$\theta_{p,k} = I_k - i_{\min,k}. \quad (48)$$

These equations are useful in finding geometrical strcuture of blade considering effect of wake onto it.

Step 5: Calculating AoA and corresponding C_L, C_D

Lift and drag data are obtained from datasheet of NACA2412 airfoil corresponding to *Angle of Attack* (AoA) calculated according to the following equation:

$$i_{k,n} = I_{k,n} - \theta_{p,k}, \quad (49)$$

Step 6: Determination of thrust coefficient

Thrust coefficient for each annular section of blade is expressed as:

$$C_T = \frac{\mathrm{d}F_N}{0.5(2\pi r\mathrm{d}r)\rho V^2}. \quad (50)$$

From (24)

$$C_{T_{k,n}} = \frac{\sigma(C_{L,k,n}\cos I_{k,n} + C_{D,k,n}\sin I_{k,n})(1-a_{k,n})^2}{\sin^2 I_{k,n}}. \quad (51)$$

Step 7: Updating a and a′ for next iteration

In turbulent wake condition, thrust obtained from momentum theory is no longer applicable. When thrust coefficient is valued more than 0.96, a and a' are updated as per empirical relationship developed by Glauert's, as follows [16].

$$a_{k,n+1} = \frac{1}{\dfrac{4P_{k,n}\sin^2 I_{k,n}}{\sigma C_L,k,n\cos I_{k,n}}+1}. \quad (52)$$

Else for, $C_{T,k,n} > 0.96$

$$a_{k,n} = \left(\frac{1}{P_{k,n}}\right)\left\{0.14+\sqrt{0.02-0.65(0.89-C_{T,k,n})}\right\}, \quad (53)$$

$$a'_{k,n+1} = \frac{1}{\dfrac{4P_{k,n}\cos I_{k,n}}{\sigma C_{L,k,n}}-1}. \quad (54)$$

If newly found values of a and a' are within the minor difference with the previously obtained one, then the process of iteration is stopped and performance parameter is calculated. In this work this difference is taken as 0.01. If the difference is more than 0.01 then process starts from equation (38) with the $n = n + 1$. As the process progresses, differences between consecutive a and a' reduce and both approach towards a solution. Assuming blades are divided into equal sections and solving the mentioned equations for each blade sections, power coefficient is calculated in the form of summation inferred from equation (35) as follows:

$$C_P = \left[\frac{8}{\lambda B}\sum_{k=k_1}^{B} P_k{\lambda_{rk}}^2\left\{1-\left(\frac{C_D}{C_L}\right)\cot I_k\right\}\right] \times\left[\sin^2 I_k(cosI_k - \lambda_{rk}\sin I_k)(\sin I_k + \lambda_{rk}\cos I_k)\right], \quad (55)$$

where, k_1 refers to first blade section of the blade.

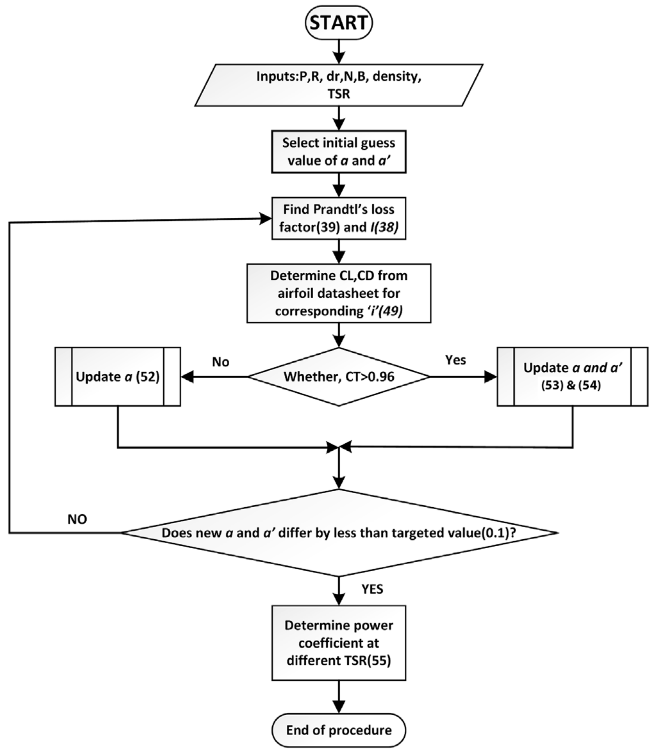


Figure 7. Design procedure for a 4 kW Small Wind turbine blade.

Step 8: Repeating the procedure

Repeat steps 5–8 until best design for the rotor blade is obtained.

This design procedure is suggested assuming a specific value of λ. Once the performance parameter is obtained at any specific value of λ, the same procedure may be adopted to find the various performance parameter at different values of λ. The whole design procedure is summarized in the pictorial form as shown in Figure 7.

6 Discussions of results

In this work, attempt has been made to design and obtaining performance characteristics for 4 kW Small Wind

Table 1. Variation in WT-blade parameter without taking wake into consideration.

r	r/R	λ_r	I	θ_p	θ_T	C
0.1	0.05	0.38	60.00	51.25	54.56	0.45
0.2	0.11	0.77	40.88	32.13	35.44	0.33
0.4	0.22	1.54	23.40	14.65	17.96	0.20
0.6	0.33	2.31	16.09	7.34	10.65	0.14
0.8	0.44	3.08	12.21	3.46	6.77	0.11
1.0	0.55	3.85	9.83	1.07	4.38	0.09
1.2	0.66	4.62	8.21	−0.54	2.77	0.07
1.4	0.77	5.39	7.05	−1.70	1.61	0.06
1.6	0.88	6.16	6.17	−2.58	0.73	0.05
1.8	1.0	7.00	5.44	−3.31	0	0.04

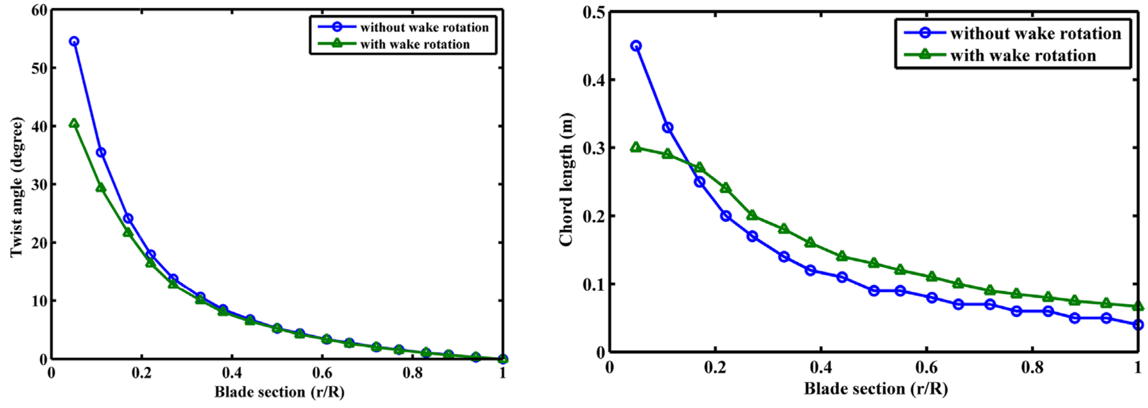
Table 2. Variation in WT-blade parameter while taking effect of wake into consideration.

r	r/R	λ_r	I	θ_p	θ_T	C
0.1	0.05	0.38	45.83	38.83	40.41	0.30
0.2	0.11	0.77	34.75	27.75	29.32	0.29
0.4	0.22	1.54	21.82	14.82	16.40	0.24
0.6	0.33	2.31	15.46	8.46	10.04	0.18
0.8	0.44	3.08	11.87	4.87	6.45	0.14
1.0	0.55	3.85	9.61	2.61	4.19	0.12
1.2	0.66	4.62	8.06	1.06	2.64	0.10
1.4	0.77	5.39	6.93	−0.06	1.51	0.09
1.6	0.88	6.16	6.08	−0.91	0.67	0.07
1.8	1.0	7.00	5.42	−1.57	0	0.06

Turbine (SWT) blade. Following the examination of several popular airfoils, NACA2412 has been taken into the consideration as most appropriate one for present work. A detailed algorithm is proposed for designing and performance analysis of a SWT-blade. From the sets of airfoil data, it is essential to find a definite *Angle of attack* at which lift: drag could be maximised so as to have maximum possible extraction of power. Therefore, blade twist angle has to varied throughout the blade length, so that each blade section faces wind velocity at "definite" *AoA*. The SWT normally operates at low *Reynolds numbers* (*Re*). Many times it may be tough to get accurate lift and drag data set at low values of *Re*. Therefore, a data co-relation has been proposed (Eqs. (1)–(5)) to obtain the lift and drag data. It is evident from Figure 3 that lift data are more significant under low *Angle of Attack* and *Reynolds numbers* at which SWT normally operates. Lift:drag data are essential for proper selection of airfoil and design of blade. Moreover, power coefficient is also dependent upon this ratio. Lift:drag is shown for several commonly used airfoils in Figure 4b for the *Reynolds numbers* and Ncrit values 50,000 and 9 respectively. It may be observed that SG6043 has the highest value of lift:drag among others whereas, SD7062 has the lowest. However, NACA2412 and SD7062 has impressive value in the low range of *AoA*, which is subject of analysis of this work. Therefore, NACA2412 has been selected as "airfoil of choice". In the process of designing, optimum geometrical blade structure *i.e.*, variation of chord length, twist angle etc. is to be determined. Such a variation of several blade parameters are tabulated in Table 1.

Twist angle and chord length are vital to determine as they ensure optimum *Angle of Attack* and uniform thrust force respectively along the entire blade length. It can be observed that chord length decreases along the blade length from the blade hub to tip. Such values of the geometrical structure of the blade can be obtained by combining forces from blade element and momentum theory. However, effect of rotating blade, which causes flow of opposite nature (wake) behind it and results in energy extraction lower than expectation has been neglected in this analysis. Hence, there is need to move for more practical approach and include the effect of wake. In this work, effect of wake has been included and results obtained are tabulated in Table 2. A comparison of both approaches is shown in Figures 8a and 8b.

It can be observed that chord length near the hub has less value if effect of wake rotation is taken into account. It has greater value from chord length (p.u) 0.18. Blade is wider as the length increases towards the tip. Twist angle is approximately equal after chord length (p.u) 0.4 for both situation, including and excluding wake rotation. There is



(a) (b)

Figure 8. Twist angle (a) and chord length, (b) variation along the blade length.

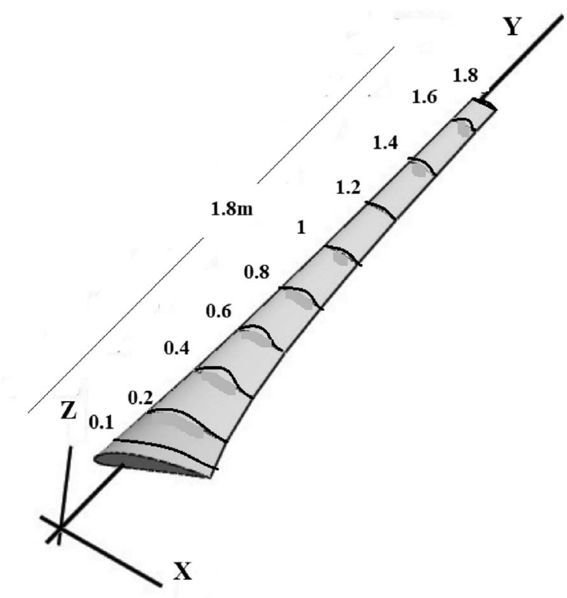


Figure 9. A typical design of blade as outcome of the work.

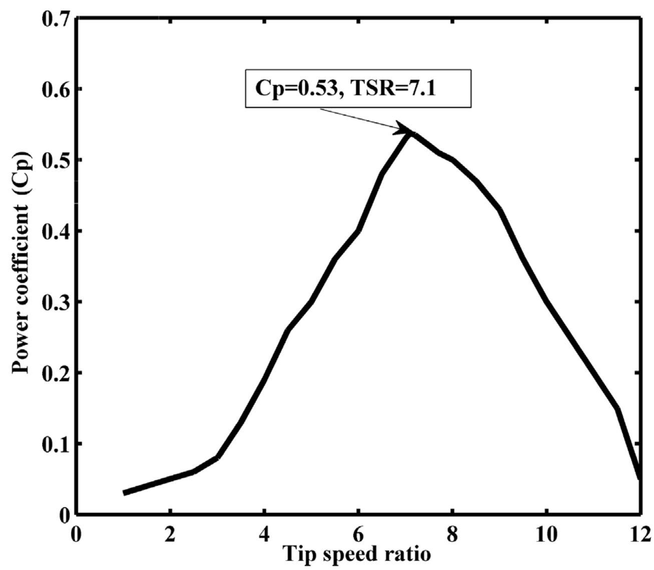


Figure 10. Performance curve of 4 kW Small Wind Turbine.

variation in other parameters as well, which are shown in the both tables. A typical layout of proposed design has been shown in Figure 9. A detailed algorithm for the design of 4 kW WT-blade has been devised stepwise. Results are obtained after several iterative procedure when solution converges to a point. Performance is investigated via graph plotted between power coefficient and tip speed ratio (λ) (Fig. 10). Maximum value of C_p is found to be at 0.53 at TSR 7.1, which is approximately in accordance with [1, 4, 13, 14].

7 Conclusion

A completely novel design procedure has been proposed for the design of 4 kW Small Wind Turbine model numerically. After examining several airfoils with their lift and drag coefficient under different Reynolds number and considering the case of Small Wind Turbine the NACA2412 airfoil has been chosen for the present work. Some of the important WT parameters are obtained and performance is analyzed. Blade is divided into several small segments for accurate design and analysis. Following the step of design procedures, section wise blade segments are obtained. Twist angle variation, chord length and other related parameters are determined considering two cases, first, excluding wake rotation, second, including it. Performance is obtained for the entire WT blade as whole. By adopting a novel design procedure performance characteristics of SWT is obtained that nearly conforms to the results of several standard experimental and analytical works in the past. Uniqueness of the proposed method is that it explicitly selects the airfoil for blade design, which is most suitable for efficient working under low range of *Angle of Attack* – a situation under which a Small Wind Turbine normally operates.

Conflicts of interests

The authors declare that they have no conflict of interest.

References

1 Wood D. (2006) *Small wind turbine analysis, design, and application*, Springer.
2 Hansen M.O. (2008) *Aerodynamics of wind turbine*, Earthscan Publication Ltd.
3 Jha D., Thakur A.N. (2017) A comprehensive review on wind energy systems for electric power generation: current situation and improved technologies to realize future development, *Int. J. Renew. Energy Res. (IJRER)* **7**, 1786–1805.
4 Manwell J.F., McGowan J.G., Rogers A.L. (2009) *Wind energy explained: theory, design and application*, John Wiley & Sons Ltd, United Kingdom, 83–139.
5 Karthikeyan N., Kalidasa K., Murugavel S., ArunKumar S. Rajakumar (2014) Review of aerodynamic developments on small horizontal axis wind turbine blade, *Renew. Sustain. Energy Rev.* **3**, 801–822.
6 Rehman S., Mehhub M., Alhems L.M., Mujahid Rafique M. (2018) Horizotnal axis wind turbine blade design methodologies for efficiency enhancement – A review, *Energies* **11**, 3, 506–512.
7 Wata J., Faizal M., Talu B., Vanawalu L., Sotia P., Rafiuddin Ahmed M. (2011) Studies on a low Reynolds number airfoil for small wind turbine applications, *Sci. China Technol. Sci.* **54**, 7, 1684–1688.
8 Gordon Leishman J. (2011) *Aerodynamics of horizontal axis wind turbines*, Springer-Verlag, Berlin Heidelberg, 1–66.
9 Muhsen H., Al-Kouz W., Khan W. (2020) Small wind turbine blade design and optimization, *Symmetry* **12**, 18.
10 Tahir A., Elgabaili M., Rajab Z., Buaossa N., Khalil A., Mohamed F. (2019) Optimization of small wind turbine blades using improved blade element momentum theory, *Wind Eng.* **43**, 299–310.
11 Amarante Mesquita A.L., Alves A.S.G. (2000) *An improved approach for performance prediction of HAWT using the strip theory*, vol. **24**, SAGE Publications, Sage UK, London, England, pp. 417–430.
12 Pinheiro Vaz J.R., Pinho J., Mesquita A. (2011) An extension of BEM method applied to horizontal-axis wind turbine design, *Renew. Energy* **36**, 1734–1740.
13 Sriti M. (2018) Improved blade element momentum theory (BEM) for predicting the aerodynamic performances of horizontal axis wind turbine blade (HAWT), *Wind Eng.* **38**, 191–202.
14 Lubitz W.D. (2014) Impact of ambient turbulence on performance of a small wind turbine, *Renew. Energy* **61**, 69–73.
15 Chaudhary U., Mondal P., Tripathy P., Nayak S.K., Saha U. K. (2014) Modeling and optimal design of small HAWT blades for analyzing the starting torque behavior, in: *Eighteenth National Power Systems Conference (NPSC)*, IEEE, pp. 1–6.
16 Glauert H. (1935) *Airplane propellers in aerodynamic theory*, Springer Verlag, Berlin, London, 169–300.
17 Lee H., Lee D.-J. (2020) Low Reynolds number effects on aerodynamic loads of a small scale wind turbine, *Renew. Energy*, **154**, 1283–1293.
18 Osei E.Y., Opoku R., Sunnu A.K., Adaramola M.S., Kyeremeh E.A. (2022) Aerodynamic performance characteristics of EYO-series low Reynolds number airfoils for small wind turbine applications, *Alex. Eng. J.*, **61**, 12301–12310.
19 Jha D., Singh M., Thakur A.N. (2021) A novel computational approach for design and performance investigation of small wind turbine blade with extended BEM theory, *Int. J. Energy Environ. Eng.* **12**, 563–575.
20 Raghavendra S., Ravikumar T., Gnaendra Reddy G., Manjunatha K., Madhusudhana S. (2020) Design of wind blades for the development of low-power wind turbines using Betz and Schmitz methods, *Adv. Mater. Process. Technol.* **8**, 808–827.